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A Constraint Equation for Climate[1]

Abstract

The IPCC's *Sixth Assessment Report (AR6)* is some 3949 pages long and is dedicated mostly to many complex details in the estimates of our future CO_2 emissions, the behavior of the atmosphere, estimations of temperatures in places without thermometers, melting permafrost, atmospheric H_2O content, ice melt, sea rise, feedback mechanisms, and so forth. The climate system is certainly complex and chaotic, but it is still subject to constraints. We will derive an important—but simple—equation of constraint. The equation will be of no use in predicting the worldwide average temperature in (say) 2060, nor will it be of use in describing the climate in any locale. Its usefulness lies in the fact that at equilibrium it must be balanced.

Introduction

IPCC's *First Assessment Report* (*FAR* 1990) acknowledged that the surface of the earth is 33ºC warmer than it would be without the greenhouse effect, a fact that was well understood for probably a century at the time. It had also been known for about a century that CO_2, H_2O, N_2O and others are greenhouse gases. *FAR* presented a simple logarithmic formula, derived from the absorption spectrum, for the "forcing"—*additional greenhouse effect*—due to increasing CO_2 concentration. The *Third Assessment Report* (*TAR*, 2001) presented three formulas for forcing—all very similar numerically—but with a 15% smaller coefficient in the logarithmic formula, and that formula has been used ever since. The atmospheric CO_2 concentration at the time of *FAR* was about 350 parts per million by volume (ppmv) (or by mole). The present (2021) concentration is about 420 ppmv. Climate models have been developed based on various assumptions about how much CO_2 will be emitted into the atmosphere and how much that CO_2 will affect the temperature directly and indirectly through feedback mechanisms. The results are expressed in the "Equilibrium Climate Sensitivity" (ECS), which is the eventual temperature increase due to a doubling of CO_2 concentration.

Complexities

The greenhouse effect in a real greenhouse was initially thought to be due to the fact that visible light would enter the greenhouse, but the glass would block outgoing infrared (IR). Fleagle[1] refers to a 1909 experiment by Johns Hopkins University physicist Robert W. Wood substituted rock salt for glass because it is transparent to IR and showed that it is just about as effective as glass in keeping the greenhouse warm. He found the greenhouse effect to be due mostly to the fact that the greenhouse is a confined space through which warm air was blocked from rising.

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In the atmosphere, the IR processes include absorption of IR by CO_2 and other greenhouse gases, collisional excitation and de-excitation, and reflection of IR by clouds back toward the ground. Heat transfer also involves evaporation, condensation, conduction, and convection, but those processes do not directly affect IR. To some people, the greenhouse effect refers only to the behavior of the greenhouse gases. In my view (and evidently in the view of the IPCC, as we shall see) it is better to use the term *greenhouse effect* more broadly to include all atmospheric phenomena that cause the IR emission to space to be less than the IR emitted by the surface. Better yet would be to use the term *atmosphere effect*, as proposed by Fleagle and Businger.[1]

IR in the absorption band of CO_2 can, in some cases, travel well less than a meter in our atmosphere before being absorbed, though IR at other wavelengths can travel tens to thousands of meters. This fact led the IPCC to construct the graph shown in the left-hand graph of Figure 1 taken from *FAR*. At about 15 μm (667 cm^{-1}) the radiation to space was reckoned as zero.

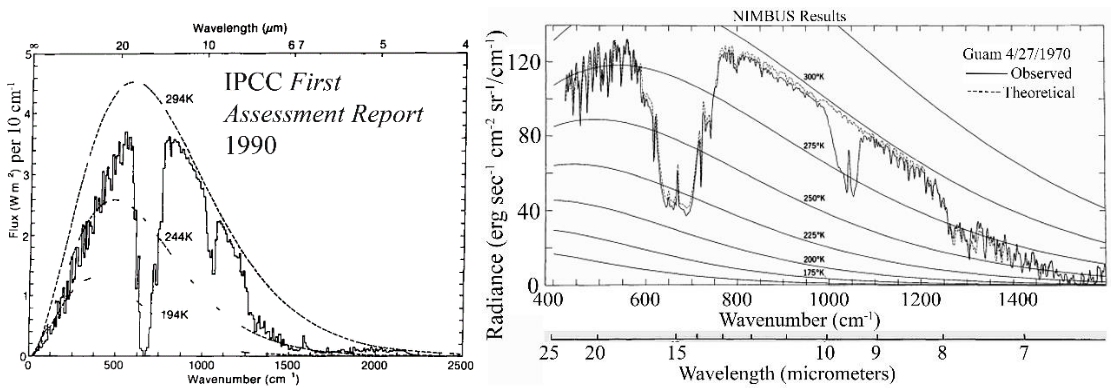

Figure 1: Left: IR radiation to outer space showing total blockage in the center of the CO_2 band, as published by the IPCC in *FAR* in 1990. Right: IR radiation to space, measured by Nimbus satellite in 1970. The theoretical curve accounts for CO_2 and h20, but not for O_3 (the dip at about 1050 cm^{-1}).

The model expressed in Figure 1, however, was naïve for neglecting high-altitude emission to space, but also in disagreement with measurements of the IR spectrum over Guam taken twenty years earlier in 1970, which showed somewhat strong radiation at that very 15-micrometer point. In the atmosphere, when a molecule absorbs IR, it can shed the energy by radiating IR in some random direction, but it can also undergo collisional de-activation, transferring the energy into kinetic energy of moving molecules. Alternatively, collisions can excite the relevant states in the greenhouse molecules which can then radiate. Temperature equilibrium demands that a small percentage of the molecules be in those excited states. Moreover, the spectral linewidths are broadened by pressure (because of proximity to other molecules) and by temperature (because of the Doppler effect). Consequently, the calculations become pretty complicated when considering the photon-molecule interactions at all altitudes and temperatures.

Our intent here is merely to show that the simple attenuation model of IR absorption is not correct. The very fact that the spectral line that is absorbed most strongly by CO_2 is also the strongest emission line to space attests to the fact that collision-induced excitation at a temperature of around 220 K is responsible for the emission to outer space.

Other complexities arise from the oceans: The Pacific Decadal Oscillation, the El Niño Southern Oscillation, ocean salinity, variations in the Gulf Stream, the temperature profile (which affects atmospheric CO_2 and H_2O) and many others. The biosphere absorbs CO_2 and sequesters some of it. IPCC's synopsis of the heat flow is shown in Figure 2, from *AR5*.[2]

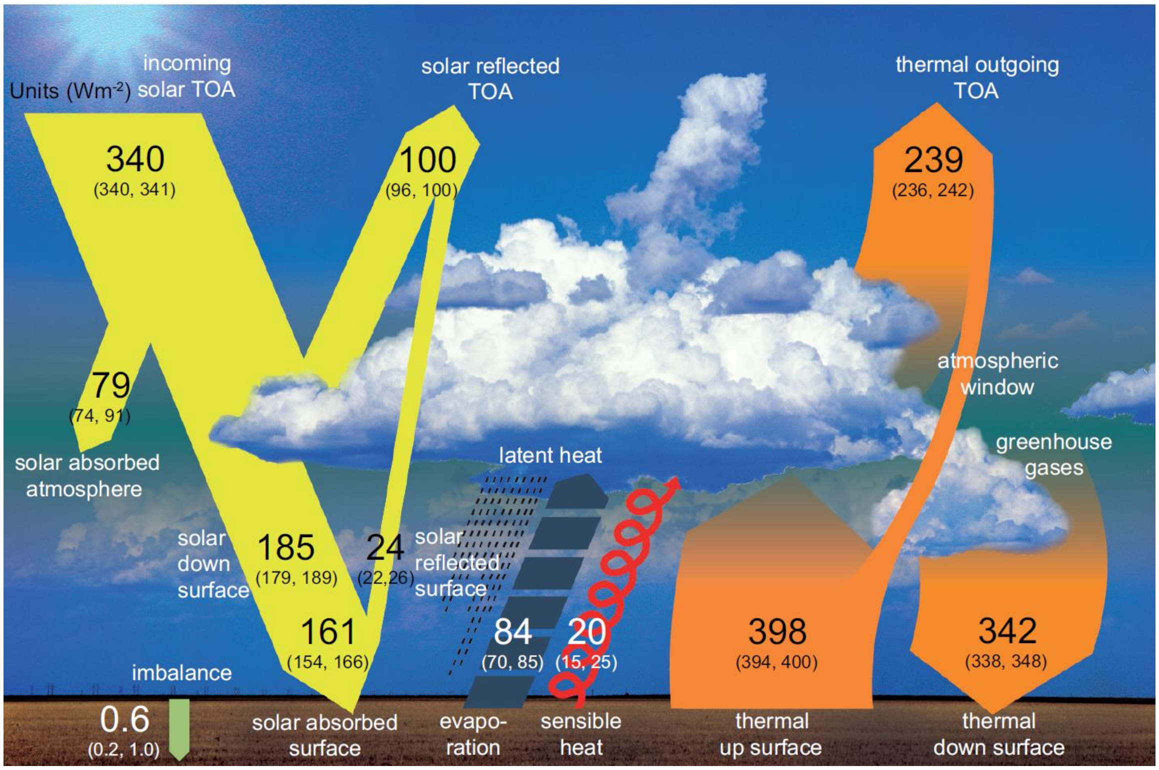

Figure 2: Heat flow diagram from *AR5*, Figure 2.11. The presumably similar figure from *AR6* has not yet been published.

Simplicities

All planets absorb heat from the sun and radiate heat back into space. At equilibrium those two amounts are equal. An imbalance leading to a steady change of an unmeasurable 0.001ºC per year would result in a 1,000ºC change in a million years, a tiny blip on the time scale of the solar system. On the other hand, an average imbalance of that magnitude must have existed on the earth as it warmed about 10ºC in about 10,000 years from the last glacial phase to the current interglacial.

One might easily wonder if this equality between absorbed heat from the sun and heat radiated to outer space holds for the earth, given the present warming. In fact, there are seasonal disequilibria. As determined by the CERES satellites,[3] during the Southern summer, sunlight penetrates many meters into the oceans, causing warming so trivial that the surface does not increase IR emission very much; moreover, the earth is nearest the sun during the Austral winter. The imbalance during that time is in the positive direction: the earth absorbs more solar heat than it radiates IR back into space. During the Northern summer, sunlight is much more likely to hit land, which warms immediately, increasing

IR emission. During this time the imbalance is in the negative direction: the earth radiates more IR energy to space than the planet absorbs from the sun.

The CERES[3] project has determined that, for the 16 years of its existence, there is a net positive imbalance of 0.7 watts per square meter, averaged over the planet, reflective of the warming during this time. By way of comparison, Figure 2 shows an imbalance at the surface of 0.6 W/m^2. Notice that the imbalance is an effect, not a cause. Compared to other values we will encounter, this imbalance is negligible.

Our first simplicity applies to all planets. At the planetary orbit, the solar intensity is I_{sun}, and the planet has its albedo (reflectivity) α. Absorbed sunlight is $I_{sun}(1-\alpha)\bullet\pi R^2$; averaged over the surface of $4\pi R^2$, the average solar intensity is $I_{sun}(1-\alpha)/4$. Equating I_{out} and I_{in}, we may write

$$I_{out} = I_{in} = I_{sun}\left(\frac{1-\alpha}{4}\right) \tag{1}$$

The state of equilibrium may seem a bit odd to consider when the subject is climate change, but climate scientists are concerned with the end game, the ECS: how much would the surface temperature eventually rise if the CO_2 concentration doubled?

At the present, the albedo of the earth is 30%, and the value of I_{out} is 239 W/m^2, as shown in Figure 2. (The CERES[3] paper assigns a value of 242 W/m^2, but to avoid confusion, we will maintain consistency with Figure 2.) Of course, it is easier to infer I_{in} from measured solar flux than to measure the average I_{out} over the whole globe for the whole year. In any case, the 0.7 W/m^2 imbalance inferred by CERES[3] (0.6 W/m^2 in Fig. 2) between the two numbers is a mere 0.3% of I_{out}. For the earth, Equation 1 is an excellent approximation at present and exact at equilibrium.

It is interesting to compare Venus with the Earth. Venus has a very high 76% albedo, so that it absorbs and emits only 156 W/m^2, although its surface temperature is a lead-melting 737 K (464ºC).

Stefan-Boltzmann & the Greenhouse Effect

The second simplicity is hiding in plain sight, but obscured by all of the attention paid to various atmosphere-surface interactions. As discussed earlier, the IPCC uses the term *greenhouse effect* to include *all* of the effects that reduce radiative flux from that emitted by the surface to that emitted to outer space. In reports prior to *AR6*, the term *greenhouse effect* was used to describe a phenomenon. In *AR6*, IPCC assigned a variable G to the effect, and asserted G = 159 W/m^2. Physicists may well be surprised that IPCC's first five *Assessment Reports* made no explicit mention of the Stefan-Boltzmann radiation law, but *AR6* mentions the law in the same paragraph (page 7-61) where they said G = 159 W/m^2.

For a perfect blackbody, the radiated power per unit area is proportional to the fourth power of the absolute temperature. The Stefan-Boltzmann radiation law, with σ = 5.67 × 10^{-8} W m^{-2} K^{-4}, is

$$I = P/A = \sigma T^4 \tag{2}$$

Solids and liquids (such as sea water) do not radiate quite as much as specified by the Stefan-Boltzmann law, so it is appropriate to multiply by an emissivity ε. For the earth, NASA[5] has found the emissivity to be about 0.95. For the present paper, sufficient accuracy can be achieved if we take the emissivity to be 1.0.

Figure 2 shows that the surface temperature of the globe is regarded to be 289 K, for which the Stefan-Boltzmann radiation law (with $\varepsilon = 1$) says that the IR emission should be 398 W/m^2, the value shown as "thermal up surface" in Figure 2. However, the radiation to space is 239 W/m^2. The difference between these two values is 159 W/m^2, which IPCC in *AR6* calls "the greenhouse effect *G*." It is this 159 W/m^2 that keeps the earth 33ºC (some would say 34ºC) warmer than it would be with the same albedo but without any greenhouse effect ($G = 0$).

With that much clarified, we may write our second simple equation:

$$G = I_{\text{surf}} - I_{\text{out}} = \sigma T_{\text{surf}}^4 - I_{\text{out}} \tag{3}$$

In other words, we can simply subtract the outbound radiation to space from the surface radiation to get the greenhouse effect. The alternate calculation of *G* from millions of absorption and emission spectral lines at all temperatures and atmospheric pressures is extremely complicated.

Let us now combine Equations 1 and 2:

$$G = \sigma T_{\text{surf}}^4 - \frac{I_{\text{sun}}}{4}\left(1-\alpha\right) \tag{4}$$

Equation 5 relates the intensity of sunlight, the albedo of the globe, the temperature of the surface, and the greenhouse effect *G*. Since we are concerned with climate change, let us find the differential

$$dG = 4\sigma T_{\text{surf}}^3 dT - \left(1-\alpha\right)\frac{dI_{\text{sun}}}{4} + \frac{I_{\text{sun}}}{4}d\alpha \tag{5}$$

Equation 5 relates changes in a particularly simple way. One can suppose that one or two variables remain constant to study the interplay between the remaining variables. The important thing about Equations 4 and 5 is that they must be balanced.

Albedo

The historical method of measuring albedo is by measuring earthshine off the moon, which has its own angle-dependent albedo. Suffice it to say that a recent measurement at the Big Bear Solar Observatory[4] noted that the present albedo is ~0.3, and that the two-decade (1998-2017) change in "earthshine-derived albedo corresponds to an increase in radiative forcing of about 0.5 W/m^2." The increase in radiative forcing is caused by a decrease in albedo, so we have in Equation 5, $(I_{\text{sun}}/4)d\alpha = -0.5$. Figure 3 of the same paper[4] gives the estimate of the albedo effect from CERES as –1.4 W/m^2. Both figures, though not equal due to the necessity of subtracting large numbers with some uncertainties, show a slight disequilibrium that is consistent with the imbalance of 0.6 W/m^2 in Figure 2.

Radiative Forcing

The term *radiative forcing* is used by the IPCC to mean any addition (positive or negative) to the greenhouse effect. The radiative forcing for CO_2 is given as

$$\Delta F = 5.35 \ln\left(\frac{C}{C_0}\right) \frac{\text{W}}{\text{m}^2} \quad (TAR,\ \ 2001 \text{ through } AR6,\ 2021) \tag{6}$$

The logarithmic form hints at why there is frequent reference to "CO_2 doubling": it's a mathematical convenience just as half-life is in discussions of radioactivity. It is likely that the 2001 formula is a bit of an overestimate, but for our purposes, it is close enough. For CO_2 doubling ($C = 2C_0$), ΔF is 3.7 W/m^2. Note also that the "Δ" symbol implies that this radiative forcing is *additive* radiative forcing. For this reason, we will henceforth use dG_{CO2} for the change in G due to CO_2 and dG_{other} for IR changes due to other causes.

A Few Sample Calculations

Equation 5 has four variables, for which the present values are I_{sun} = 1366 W/m^2; α = 0.3; T_{surf} = 289 K; and G = 159 W/m^2.

Let us assume for a moment that sunlight and the greenhouse effect both remain the same (dI_{sun} = 0 and dG = 0). Then we have a relationship between the albedo and the surface temperature:

$$4\sigma T_{surf}^3 \, dT = -\frac{I_{sun}}{4} d\alpha \tag{7}$$

If the albedo increases by 2%, the incoming (hence the outgoing) radiation is reduced by about 6.8 W/m^2. That reduction comes from a drop of 1.2ºC in surface temperature. Very likely, a drop in albedo caused the Year Without a Summer in 1816 after the eruption of Mount Tambora in 1815. Volcanic ash reflected sunlight, and the increased albedo caused a decrease in temperature.

The amount of sunlight reaching our orbit has been known since the late 1800s, but more precise measurements awaited our ventures to the upper atmosphere and to nearby space. Estimates of variations in I_{sun} for the last several centuries vary from about 1 W/m^2 to about 10 W/m^2, both less than 1% of I_{sun} of 1366 W/m^2. With constant albedo and constant G, a 1% change in I_{sun} would change the surface temperature by about 0.6ºC.

If sunlight does not vary (dI_{sun} = 0), and the albedo of the earth remains constant ($d\alpha$ = 0), then the left side of Equation 5 is zero, and we have a simple relationship between changes in the greenhouse effect (that is, radiative forcing) and surface temperature.

$$dG = 4\sigma T_{surf}^3 \, dT \tag{8}$$

That is, if sunlight and albedo remain constant, then any change in surface emission must be matched by an increase in greenhouse effect, and conversely.

Example 1: Warming from 1750 to present

It is interesting to look at two lines of evidence, both dating from 1750 to the present. Figure 3 shows the total anthropogenic forcing is about 2.3 ± 1.1 W/m^2. Under the conditions of a constant sun and constant albedo, the temperature rise should be 0.4ºC.

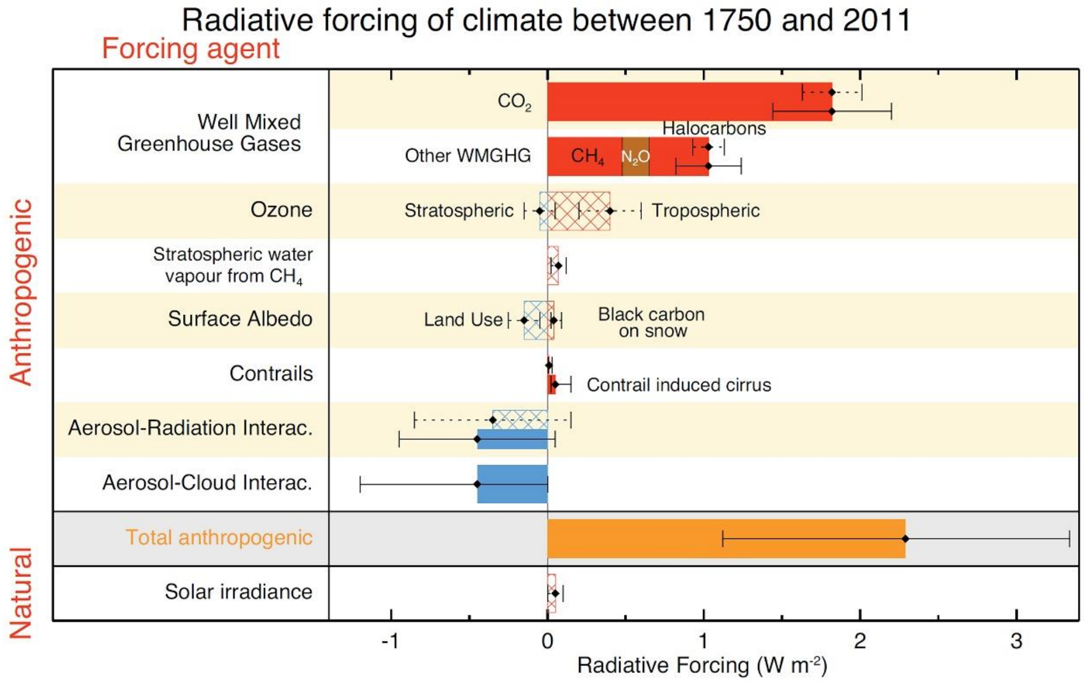

Figure 8.15 | Bar chart for RF (hatched) and ERF (solid) for the period 1750-2011, where the total ERF is derived from Figure 8.16. Uncertainties (5 to 95% confidence range) are given for RF (dotted lines) and ERF (solid lines).

Figure 3: Radiative forcing since 1750, as reported in *AR5* p. 697. The mean total anthropogenic forcing is about 2.3 W/m^2, and that due to CO_2 is 1.75 W/m^2. "RF" = Radiative Forcing; "ERF" = Effective Radiative Forcing (*i.e.*, RF at equilibrium)

Figure 4 shows IPCC's *AR5* graph of temperature since 500; the rise since 1750 has been about 1.2ºC. The Stefan-Boltzmann radiation law tells us that the surface should emit 6.6 W/m^2 more than it did in 1750.

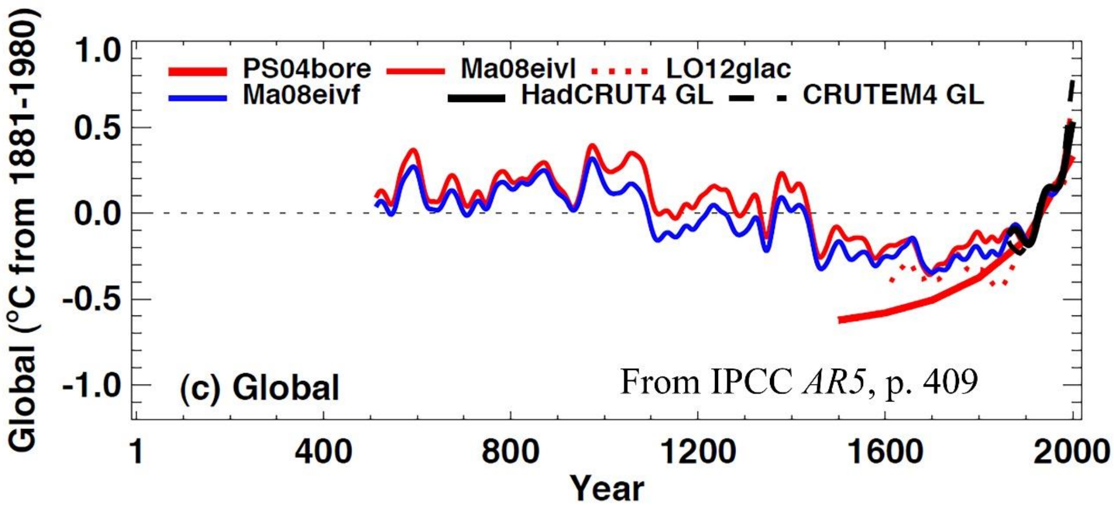

Figure 4: The global temperature record from 1000 to the present, showing about 1.2ºC rise from 1750 to present (from *AR5*).

The radiative forcing from CO_2 since 1750 is enough cause the surface temperature to rise by 0.3ºC (total anthropogenic by 0.4ºC), yet it actually rose by 1.2ºC. The radiative forcing (Fig. 3) since 1750 is about 2.3 W/m^2, yet the surface radiation has increased by 6.6 W/m^2. To balance Equation 5 requires that some combination of an increase in G_{other}, an increase in I_{sun}, and a decrease in albedo to account for the 4.3 W/m^2 difference.

AR6 (p. 1-41) says, "Like all previous IPCC reports, *AR5* assessed that total radiative forcing has been positive at least since 1850–1900, leading to an uptake of energy by the climate system, and that the largest single contribution to total radiative forcing is the rising atmospheric concentration of CO_2 since 1750." The term *total radiative forcing* "over a given time interval (often since 1750) represents the sum of positive drivers (inducing warming) and negative ones (inducing cooling)." On page SPM-40, changes in aerosols (hence in albedo) are included in the term, so it is entirely unclear whether IPCC has properly accounted for the 4.3 W/m^2 imbalance in Equation 5.

Example 2: Warming from CO_2 doubling

Consider the consequences of doubling the CO_2 concentration. By IPCC's estimate, doubling CO_2 should have two consequences: a radiative forcing of 3.7 W/m^2, and a most probable temperature rise in the vicinity of 3ºC. The forcing, by itself, should cause a temperature rise of 0.68ºC. The projected temperature rise of 3ºC means that the surface should radiate 16.5 W/m^2 more than it does now. Of course, the IPCC's projections assume constant solar intensity, so the additional 12.8 W/m^2 must be accounted for by some combination of increased greenhouse effect from other gases (primarily H_2O), increased IR reflection from the bottoms of clouds, and a decrease in albedo.

Example 3: Glacial-to-interglacial warming

According to data from ice cores at Vostok in Antarctica, near the end of the last glacial cycle about 18,000 years ago the CO_2 concentration was about 190 ppmv, versus the present 260 ppmv 8,000 years ago when the earth had warmed by 10ºC. According to the “forcing” formula (Eq. 6), CO_2, is responsible for about 1.7 W/m^2. By contrast, the Stefan-Boltzmann radiation law tells us that the additional IR emission from the surface has been about 55 W/m^2. This means that some combination of positive dG_{other}, dI_{sun} and decreased albedo must account for 53.3 W/m^2.

Conclusions

Three facts allow us to construct a simple formula that serves as a constraint on climate models: the heat radiated by the earth to outer space equals the heat absorbed by the sun; the heat radiated by the surface is given by the Stefan-Boltzmann radiation law; and the greenhouse effect equals the difference between the IR emitted by the surface and the IR emitted to outer space. The “radiative forcing” due to CO_2 adds to the greenhouse effect. If the concentration of CO_2 doubles, it increases the greenhouse effect (directly) by 2.3%. Three examples—1705 to present, IPCC’s estimate of temperature increase due to CO_2 doubling, and glacial-to-interglacial warming—show that the radiative forcing due to CO_2 increase falls far short of handling the increased surface radiation.

References

[1] Robert G. Fleagle and Joost A. Businger, The “Greenhouse Effect,” *Science*, 12 December, 1975

[2] All IPCC reports are available as PDFs at https://www.ipcc.ch/. *The Sixth Assessment Report* is presently (October 2021) incomplete.

[3] Earth’s Energy Budget at https://ceres.larc.nasa.gov/science/

[4] Goode, P. R., Pallé, E., Shoumko, A., Shoumko, S., Montañes-Rodriguez, P., & Koonin, S. E. (2021). Earth's albedo 1998–2017 as measured from earthshine. *Geophysical Research Letters*, 48, e2021GL094888. https://doi. org/10.1029/2021GL094888

[5] https://www.jpl.nasa.gov/images/nasa-spacecraft-maps-earths-global-emissivity